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M.Sc. (Sem. - IV) Examination September - 2025

Mathematics (PGMTH : 4043)

Advanced Integral Transforms-II

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Follow usual notations and conventions (3) Figures to the right indicates marks (4) Answer all questions	Student's Signature

Q. 1 Attempt Any Two: **14**

(a) Define Fourier transform and Inverse Fourier transform.

Find Fourier transform of the function $f(x)$, $f(x) = \begin{cases} x^2; & \text{if } |x| \leq a \\ 0; & \text{if } |x| > a \end{cases}$

(b) If $F[f(x)] = f(s)$, then prove that

i. $F[f(x-a)] = e^{-ias} f(s)$

ii. $F[x^n f(x)] = (i)^n \dfrac{d^n}{ds^n}[f(S)]$

(c) Find Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1-x^2; & \text{if } |x|<1 \\ 0; & \text{if } |x| \geq 1 \end{cases}$

Hence deduce that $\int_0^\infty \dfrac{\sin(s) - s\cos(s)}{s^3} \cos\left(\dfrac{s}{2}\right) ds = \left(\dfrac{3\pi}{16}\right)$.

Q. 2 Attempt Any Two: **14**

(a) If $F_s[f(x)] = f_s(s)$ and $f_c[g(x)] = g_c(s)$, then prove that

i. $\int_0^\infty |f(x)|^2 dx = \int_0^\infty |f_S(s)|^2 ds$

ii. $\int_0^\infty |g(x)|^2 dx = \int_0^\infty |g_c(s)|^2 ds$

(b) In usual notations, prove that

$F[(f * g)(x)] = F[f(x)].\ F[g(x)] = \bar{f}(s)\, \bar{g}(s)$

(c) Find $f(x)$ if its Fourier cosine transform is $\dfrac{1}{1+s^2}$.

Q. 3 Attempt Any Two: **14**

(a) Define Fourier Sine and Cosine Transforms.

i. If $F_c[f(x)] = f_c(s)$, then prove that $F_c[f(ax)] = \dfrac{1}{a} fc\left(\dfrac{s}{a}\right)$.

ii. If $F_s[f(x)] = f_s(s)$, then prove that $F_s[xf(x)] = -\dfrac{d}{ds}[F_c(S)]$

(b) Find Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate Fourier Sine transform of $xe^{-a^2x^2}$.

(c) Express following function as Fourier integral and hence evaluate
$\int_0^{\infty} \frac{\sin\lambda \cos(\lambda x)}{\lambda} d\lambda$, Where $f(x) = \begin{cases} 1; & if\ |x| \le 1 \\ 0; & if\ |x| > 1 \end{cases}$

Q. 4 Attempt Any Two: **14**

(a) Define (i) Finite Fourier Sine transform (ii) Finite Fourier cosine transform
Find finite cosine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2$, $0 < x < \pi$.

(b) Use finite Fourier Cosine transform to solve $\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2}$ $(0 < x < \pi, t > 0)$,
i. $\frac{\partial v}{\partial x} = 0$, for $x = 0$; $x = \pi$ for $t > 0$
ii. $v = f(x)$, for $t = 0$; $0 < x < \pi$

(c) Find finite sine and cosine transform of $f(x) = 2x$; $0 < x < 4$.

Q. 5 Attempt Any Two: **14**

(a) Solve using Fourier Sine transform,
$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, subject to boundary conditions
i. $u(0, t) = 0$
ii. $u(x, 0) = e^{-x}$ and $u(x, t)$ is bounded.

(b) Use Fourier transform to determine the displacement $y(x, t)$ of an infinite string given that string is initially at rest and the initial displacement is $f(x)$ for $-\infty < x < \infty$. Show that solution can be put in the form
$y(x, t) = \frac{1}{2}[f(x + ct) + f(x - ct)]$.

(c) The temperature u in semi finite rod is given by,
$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, $0 \le x < \infty$, subject to the conditions
i. $u = 0$ when $t = 0$, $x > 0$
ii. $\frac{\partial u}{\partial x} = \mu$ when $x = 0$
iii. $\frac{\partial u}{\partial t} \to 0$ as $x \to \infty$
Determine temperature using Fourier Cosine transform.